Developmental research has been troubled by a fundamental contradiction between theoretical assumptions and tools for analysis. On the one hand, theories of development such as Piaget's (Piaget, 1957; Piaget, 1975) and the various neo-Piagetian theories (Case, 1985; Fischer, 1980; Halford, 1987; Siegler, 1981) posit intricate growth processes that predict complex patterns of development, such as stage discontinuities and equilibration. On the other hand, methodological tools for describing development have long limited analysis to a small set of simple growth patterns, mostly involving linear or monotonic change. Development is more complex and interesting than that! If the theories are at all correct, development should have diverse, complex shapes.

Developmental theories require more complex, sophisticated tools for analysis than have been used in the past, and happily such tools are now available. Methods based on nonlinear dynamics provide powerful ways of representing and analyzing processes of change and predicting patterns of development. These dynamic methods work naturally with developmental theories to produce a new powerful combination of carefully defined theoretical models and strong analytic methods. With these new tools, developmental scholars can begin for the first time to capture the complexities of the diverse shapes of human development.

For many years scholars have been calling for the use of systems theory and nonlinear dynamics in the behavioral sciences (Sameroff & Chandler, 1975; von Bertalanffy, 1968), but the tools needed for general develop-
mental analysis have been missing. As a result, work based on dynamic systems concepts has focused primarily on broad philosophical inquiries, with only a few investigators finding ways to build dynamic analyses of specific psychological problems, such as motor coordination (Thelen & Smith, 1994). Many developmental researchers have recognized the potential relevance of nonlinear systems to development, but few have found ways of using nonlinear concepts productively in their research and theory.

Recently this situation has changed dramatically, partly because the general availability of powerful computers has made dynamical tools more accessible and partly because developmental researchers have pioneered ways of using dynamical tools to analyze development (Fischer & Rose, 1994; Port & van Gelder, 1995; van der Maas & Molenaar, 1992; van Geert, 1994; Smith, chapter 9, this volume). The most promising of the newly available methods combine growth analysis with nonlinear dynamical theory (popularly known as the theory of chaos and catastrophe). These tools facilitate joining theoretical analyses of growth processes with mathematical models of growth curves and statistical tools for growth analysis. Through this combination they support much stronger connection between theory and data than has been previously possible in most developmental work. After outlining the conceptions of growth and development embodied in these tools, we spell out a dynamic model of growth processes for an illustrative domain and show how those processes can be represented in nonlinear dynamic models. The domain for illustrating use of the tools is development of concepts of self in important relationships, based on a study of adolescents in Korea (Kennedy, 1994).

NATURE OF GROWTH AND DEVELOPMENT

Conventionally, growth and development have been defined as systematic or directional changes (usually increases) in some measure of behavior or physical state, with the change typically conceived as linear or monotonic (Rasch, 1966; Willett, Ayoub, & Robinson, 1991; Wohlwill, 1973). Dynamic modeling provides broader, more powerful definitions based on models of change processes. Growth is defined as any kind of systematic change, including not only linear increase and decrease but also complex patterns such as oscillation between limits or increase occurring in successive spurts and drops. The systematic nature of the growth is described by an equation (or set of equations) that is derived from particular concepts of growth processes, and the equation predicts a family of growth curves, often of diverse shapes.

Figure 7.1 presents four different examples of growth curves generated by growth models. All four curves show systematic growth, but only one shows linear or monotonic growth. Grower \( w \) is produced by a simple logistic growth model, which generates the S-shaped curve that typifies much simple growth. It is monotonic, growing smoothly upward throughout its trajectory. Many simple growth processes involve this form of growth, where the change at a given moment is based on three parameters: the prior level of the grower, the growth rate of the system, and a limit on the system's eventual level, called the carrying capacity. The model is called logistic because the equation includes log values (squares or higher powers of the grower's level).

On the other hand, Growers \( x \), \( y \), and \( z \) involve fundamentally nonlinear growth processes defined by a single hierarchical model of five related tasks each growing through five distinct levels or stages. Despite the great differences in shape, all three curves derive from exactly the same general equations. Only the values of the parameters for the growth processes are different. Thus the "same" growth processes produce nearly
monotonic growth (Grower x), growth with spurts and drops (Grower z), and fluctuating growth that does not show general increase (Grower y). The model that generates these growth curves is one that we constructed for development of self-in-relationships in Korean adolescents.

In growth modeling, *development* is often used synonymously with *growth*. When the terms are used more precisely, *growth* refers to any systematic change defined by a model of change processes, whereas *development* refers to change that has a clear directionality, even when there are decreases or fluctuations along the way, as shown by Grower z, for example. More important than the precise definitions of terms, however, is the general point that growth is defined by systematic change processes that can be expressed mathematically, not by the shape of any one particular curve.

### VARIABILITY IN ACTIVITY AND GROWTH

The starting point of this framework is the extensive variability that human beings naturally show in their action and thought. The actual behavior of real people does not fit conventional beliefs in monolithic competence! People act at multiple levels of skill, not only one level, even for a narrowly defined domain. Moreover, individual growth curves seldom show simple monotonic increases (Fischer, Knight, & Van Parys, 1993; Thelen & Smith, 1994; van der Maas & Molenaar, 1992; van Geert, 1994).

Traditional research methods hide the nonlinear nature of development to produce the illusion of monotonic growth by minimizing variability. In this way, social science researchers make action and thought seem constant or stable instead of variable. Most commonly, they combine many related behaviors in a summary score, thereby minimizing variability by summing many items (Horn & Hofer, 1992; Sternberg & Powell, 1983). In addition, they typically include only items that show simple distributions of scores and smooth growth functions, deleting items with more complex distributions or functions. Individual traits or competences are then treated in terms of the summary scores, which are standardized relative to ranking in a distribution of scores for a reference group. Well-known examples of this procedure are standardized intelligence and personality tests, which were devised through elimination of items that do not show monotonic growth. As any teacher knows, children's performances are much more variable than their scores on intelligence or personality tests (Gardner, 1983).

Even with individualized assessment, simple transformations of observed behavior can hide variability and force data into apparently monotonic growth functions. One example is vocabulary in late infancy, which is routinely measured by cumulating a child's words (Anisfeld, 1984; Bates, Thal, & Janowsky, 1991; Brown, 1973; Dromi, 1989; Patterson, 1980). The standard view based on this summing procedure is that vocabulary shows a steady increase, perturbed by at most the existence of one period of rapid increase during an early growth phase (Reznick & Goldfield, 1992). Vocabularv is typically measured in ways that make growth seem much more regular than it actually is. Researchers use a remarkably distorting procedure, measuring not actual vocabulary use but a fictional entity called *cumulative vocabulary*. Once a word has been judged to be present in a child's lexicon (based on one or a few uses), it is scored as present for the rest of the study, whether or not the child ever uses the word again. Real children learning to speak commonly use a word a few times and then stop using it.

With cumulative vocabulary, growth must necessarily move steadily upward, and apparent variability is minimized, as illustrated in Fig. 7.2 for a toddler named John. In a longitudinal study of early language development in three children, Corrigan (1977, 1983) analyzed the actual vocabulary used per session and also scored cumulative vocabulary. Cumulative
vocabulary grew smoothly, as reflected in the monotonic curve for John in Fig. 7.2, whereas actual word use was much more variable, showing nonlinear upward growth. Cumulative vocabulary masked the variability in actual word use.

In a similar way, most developmental research has used measures that make growth appear smooth when it is in fact variable and nonlinear. Even physical growth is nonlinear—not smooth—but graphs of height and weight in developmental textbooks and pediatrician’s offices make it seem that smooth growth is the norm. The few investigators who have examined individual physical growth have found that, just like behavioral growth, it shows highly variable functions, not smooth ones. Individual growth of standard physical measures such as height, weight, head circumference, and folds of fat, for example, shows clear fits and starts, not smooth increases (Lampl & Emde, 1983; Lampl, Veldhuis, & Johnson, 1992).

Sometimes scholars are disheartened by the evidence for individual variability and nonlinear growth curves. They find the variation overwhelming because it seems to increase the difficulty of conceptualizing and explaining development. However, we demonstrate exactly the opposite—that analyzing variability provides powerful new tools for research and assessment. Through attending to the variability and capturing much of it via dynamic analysis, researchers can find potent new ways to detect order in development. Two topics where attending to variability has already demonstrated this power are the question of stages in psychological development and the debate about development of self in Asian cultures.

**CONSTRUCTIVE DYNAMICS**

Specification of processes of growth and development facilitates building dynamic models of variability and change. The nonlinear dynamic tools we describe can be used with a wide range of different concepts of process (even those assuming primarily monotonic growth), but they require the specification of an explicit model. We begin with description of the growth processes posited by the dynamic skills approach to development (Fischer, 1980; Fischer & Farrar, 1987; Fischer & Rose, 1994), which serve as the conception upon which our illustrative dynamic models are built.

**Many Components, Many Shapes**

People construct their activities from many different components and influences, which come together to produce a single activity. Every activity is composed of many constituent parts, which include the effects of the person’s body (hands, arms, eyes, posture, etc.) and of the objects, events, and people acted upon, in addition to the control structures coordinating all the pieces. Emotions, cultural meanings, and situational specifics all contribute to each activity as well. Of course, all the components and influences cannot be included in any single study, but with the dynamic perspective, each study of a few factors is framed in terms of a dynamic combination of multiple factors producing a variable outcome.

A dynamic model of growth defines a basic growth function for each specified component, which is called a grower (van Geert, 1994). Influences between growers are represented by connections between growth functions. These connections vary from strong to weak to nonexistent, and they can either promote or reduce growth processes (support or competition, respectively). The curve for a grower is a result of not only its basic growth function but also its connections with other growers.

There are many components in each activity, and growth curves can change dramatically as components shift their contributions. As a result, each person shows a wide range of growth functions, as illustrated in Fig. 7.1. There are many shapes to growth and development, including clear stage-like change, monotonic growth, and chaotic variation. Nonlinear dynamic modeling provides tools for capturing this array of shapes.

**Developmental Webs: Domains, Fractionation, and Integration**

The conventional definition of development as linear or monotonic goes with the assumption that developmental change occurs in a single sequence of steps, like a ladder. A better metaphor for development—one that captures the inherent variability—is a web or bush, as shown in Fig. 7.3 (Bidell & Fischer, 1992). Each grower develops along multiple strands simultaneously, with each strand representing a different domain of activity. These domains constitute one key component in dynamic skill models of development. By most criteria, the strands or domains are independent of each other, although there can be weak links between them, such as support and competition. Each strand includes its own subset of growers (component skills) that are strongly connected in hierarchical patterns, such as an earlier grower being a prerequisite for a later one.

In this way different activities are naturally fractionated—organized into separate strands that require active coordination to be combined or integrated. According to this process model, “the mind” is not unitary or whole but is composed of many independent parts. In specific situations, some of these strands can be combined to form coordinated skills in a new strand. This process of hierarchical coordination is fundamental in most of the tasks described by Piaget and other researchers on cognitive development.
In early reading, strands include recognizing letters and combining them into written words, analyzing the sounds of words, and understanding the meanings of words (Knight & Fischer, 1992). In development of social interactions, strands include positive interactions (nice, good, pleasant), negative interactions (mean, bad, unpleasant), and interactions that combine positive and negative (nice and mean, good and bad, pleasant and unpleasant; Fischer & Ayoub, 1994). Distinct tasks can also define strands, because task effects are one of the most powerful contributors to activity (Fischer, 1980; Flavell, 1982). For example, for Piagetian formal—operational thought, two separate strands involve the balance beam task and the task for combining kinds of objects such as buttons (Martarano, 1977; Neimark, 1975). For the development of concepts of self in important relationships (mother, father, best friend, sibling, teacher, etc.), each relationship constitutes a separate strand initially. With development, adolescents commonly relate some of these strands to each other, as we demonstrate in the study of self-in-relationships in Korea.

Each strand in a web shows strong developmental orderings, with some skills developing before others. Most of the sequences documented in developmental research constitute one such strand in a web. Each strand itself has components that develop through successive steps in a sequence. Later components are constructed on earlier ones hierarchically, including and depending on them. This is one of the strongest kinds of connection among components. For example, in reading, a child first knows the meaning of a common word, then learns to recognize the word in written form, and finally learns to write the same word himself. In mathematics, a girl first enjoys learning to do simple multiplication tasks in her second-grade class (2 times 3 equals 6, 5 times 4 equals 20), later describes herself as liking multiplication problems, and still later characterizes herself as interested in mathematics. For self-in-relationships, a Korean boy first sees himself as being well taken care of and loved by his mother, later describes himself as secure with his mother and supported by her, and still later portrays himself as confident because of the security he derives from his mother's love and support.

A common kind of hierarchical connection occurs when two strands are integrated into a new single strand. Across strands, developmental orderings are weak or nonexistent unless people coordinate the strands. Examples of coordination are commonplace, especially for tasks or contents that eventually become part of a coherent domain. With reading, children combine letters with analysis of the sounds of words to form skills for reading words. In mathematics, they combine enjoyment of doing multiplication problems with liking to talk about numbers to form a concept of interest in mathematics. For self-in-relationships, a boy combines feeling loved by his mother with knowing she will help him deal with problems at school to construct a concept of security with her. In Fig. 7.3 coordination is indicated by strands coming together, and differentiation is indicated by a strand dividing into two or more separate strands.

Even when strands are independent by most criteria, they still can show common growth patterns, such as concurrent discontinuities or "stages," as indicated by the box in Fig. 7.3. In the concurrent zone marked by the box, most strands show some type of discontinuity—change in direction, branching, or integration. These discontinuities typically mark processes of reorganization in each strand, but the concurrence across strands does not mean that the strands share strong connections. Such concurrence is commonly taken to indicate the existence of some single integrated cognitive structure, but dynamic models shows that such an inference is not justified. Concurrent discontinuities typically reflect parallel independent growth processes that have similar dynamic properties (Fischer & Rose, 1994; Grossberg, 1988; Rakic, Bourgeois, Eckenhoff, Zecovic, & Goldman-Rakic, 1986; Rumelhart & McClelland, 1988). They do not require a common cognitive structure!

With a specific model of growth processes and connections, researchers can know when and how to combine items to form a composite measure. When strands are candidates for coordination and thus strong connection, as in the several role relationships assessed in the self-in-relationships study, measures of each strand can be combined for that domain. How-
ever, the combinations must be done based on the nature of the growth processes, so that combining scores does not disguise the dynamics of growth and lead to misleadingly smooth growth functions.

Developmental Ranges: Optimal and Functional Levels

The metaphor of the web captures the general pattern of development across distinct domains, but it needs to be qualified in one important way. For each strand in the web, a developing child is not at a single point but instead acts dynamically across a range or zone for that strand, as shown in Table 7.1. Depending on factors such as motivation, emotion, and contextual support, a child will act at higher or lower steps along the strand at a given time. This variation, called the developmental range, occurs powerfully in every domain where we have searched for it (Fischer, Bullock, Rotenberg, & Raya, 1993). Developmental range is related to Vygotsky's (1978) concept of the zone of proximal development, but developmental range is more precisely defined, with specification of the processes controlling the variation. Vygotsky's zone includes multiple processes besides developmental range, such as scaffolding (Bruner, 1982; Wood, 1980) and collaboration (Fischer & Granott, 1995; Granott, 1993).

Developmental range is defined as the distance along a developmental scale (an index of a strand in the web) between optimal and functional levels. Under optimal conditions, when people are familiar with a domain, motivated to perform well, and given contextual support and priming for high-level activities, they act in complex ways, showing a relatively advanced upper limit to their skill, called their optimal level. Under ordinary conditions, without familiarity, motivation, contextual support, and priming, people act in less complex ways, showing a less advanced upper limit, called their functional level. Indeed, even when people are familiar with a domain and motivated to perform well, they show a lower (functional) level so long as contextual priming is absent, as we will illustrate for the development of concepts of self-in-relationships in Korea. A process model of growth needs to include this kind of variability as well as the independent strands in the developmental web.

Optimal and functional levels differ not only in their complexity but also in the shapes of their growth curves. Optimal levels reflect the upper limit of a person's skills under conditions that support high-level performance, and they therefore reflect the stagelike jumps in skill that occur as people move into a new, qualitatively different hierarchical level of skill. The discontinuous changes in strands in the concurrent zone in Fig. 7.3 indicate these stagelike jumps in optimal performance, and the levels listed in the left hand column of Table 7.1 indicate the points of discontinuity as well. Functional levels, on the other hand, reflect a person's upper limit under ordinary, low-support conditions, and they often show monotonic growth, with no evidence for stagelike jumps. When functional levels do show jumps in performance, they tend to be inconsistent across domains. Previous research has supported this distinction, showing concurrent stagelike changes for optimal levels and not for functional levels (Fischer, Pipp, & Bullock, 1984; Kitchener, Lynch, Fischer, & Wood, 1993).

In this way the shapes of development vary dynamically between nonlinear, stage-like change and monotonic growth depending upon whether people are performing at optimal or functional levels.

Weak Connections Between Domains

Besides the strong connections between growers in a hierarchy, growers can affect each other in smaller ways, both within and between strands. These weak connections are often difficult to detect except when they cumulate, either from their repeated action over long periods or from multiple connections converging at the same time. These connections between growers turn out to be powerful determinants of the shapes of growth, although they have been neglected in other efforts to build models of development, such as parallel-distributed-processing networks (Mareschal, Plunkett, & Harris, 1995; Rumelhart & McClelland, 1988; Shultz, Schmidt, Buckingham, & Mareschal, 1995), even the most richly detailed ones (Grossberg, 1988).

One common type of weak connection is competition, in which growth in one component or strand interferes with growth in another. For example, concentration on one relationship can compete with growth processes in another relationship, as when an adolescent spends great energy and time building a romantic relationship and thus has little time to spend...
Building or sustaining a relationship with a best friend. In another kind of competition, identifying the ideal self as independent can interfere with maintaining a close relationship with parents.

Another important type of weak connection is support, in which growth in one component or strand facilitates growth in another. For example, adolescents’ construction of a romantic relationship can promote understanding of their parents’ love and partnership. In another kind of support, building a friendship with a fellow student or coworker can further a friendship with their siblings.

Constructing a model of growth processes requires defining the several types of strong and weak connections between growers. The connections contribute powerfully to explaining both the diversity of developmental pathways across people and the variations within each person.

DEVELOPMENT OF SELF-IN-RELATIONSHIPS
IN KOREA

Building a dynamic model of development requires not only specifying the nature of developmental processes in general, as with the developmental web and range, but also tying down these concepts for some particular domain of activity. Starting with a specific study greatly facilitates building the process model by defining the specific activities and contexts to be modeled. Also, the results of the study can be tested against the outcomes of the model. To illustrate dynamic analysis, we focus on a study of the development of self-in-relationships in Seoul, Korea (Kennedy, 1994).

Scholars have often claimed that because Far Eastern cultures are collectivist, people in those cultures have no clear self-concept comparable to that of individualistic Western peoples (Markus & Kitayama, 1991; Triandis, 1989; Wallbott & Scherer, 1995). When Western tests and procedures are used to assess Eastern self-concepts, the self-descriptions obtained have been judged to be primitive and simple, like those of Triandis, 1989; Wallbott & Scherer, 1995). When Western tests and procedures are used to assess Eastern self-concepts, the self-descriptions obtained have been judged to be primitive and simple, like those of American adolescents. These complex self descriptions will produce much more complex self descriptions, comparable in developmental level to those of American adolescents. These complex self descriptions will be found empirically, but they will also be captured by straightforward variations in the dynamic model.

Method

Seventy-two middle-class students in grades 8 to 13 (6 boys and 6 girls from each grade) were interviewed individually at their school by native Korean research assistants using the Self-in-Relationships Interview (SIR) in a single session that lasted 1½ hours. The SIR is designed to assess adolescents’ and young adults’ conceptions and feelings about themselves in various relationships. Based on an instrument originally developed by Harter and Monsour (1992), it differentiates assessments for low and high support as well as assessments for each hypothesized skill level. Students were asked to describe themselves in several social contexts or role relationships and then to place their self-descriptions on a diagram and relate them to each other.

The SIR was designed to assess both functional and optimal levels of self-understanding with low and high support conditions, respectively. The low support condition assessed a participant’s functional level through the traditional “spontaneous” assessment advocated by McGuire and McGuire (1982). Participants were asked simply to describe what they were like with each of the following people: father, mother, brother/sister, boyfriend/girlfriend, best friend, other friends, and in school. They were also asked to describe the “real me.” After describing themselves in these relationships, students were asked to think about any aspects of themselves that seemed to go together, as well as any attributes that were opposites.

The high support condition of the SIR, which followed the low support condition, provided visual and social support to help participants keep in mind and relate their descriptions. Participants were asked to provide several characteristics of themselves with each of the designated people and situations, as well as the “real me.” They wrote each description on a self-sticking notepaper and indicated whether it was positive, negative, or of mixed valence. They then arranged the descriptions on an 18-in circular self-diagram by placing each self-description in one of three concentric circles that ranged from most important in the inner circle to least important in the outer circle, as shown in Fig. 7.4. Each student grouped descriptions together on the diagram and indicated relations between groups or individual descriptions. Levels of understanding were

of these jumps may be delayed, however, relative to American youths, who are not restrained from focusing on themselves in conversations. Likewise, the gap between optimal and functional levels will be larger in Korean youths than in American youths, because of the Korean devaluation of focusing on self in conversations. Not only will these differences be found empirically, but they will also be captured by straightforward variations in the dynamic model.

7. TOOLS FOR ANALYZING THE MANY SHAPES OF DEVELOPMENT
assessed through probes for eliciting skill levels for representational systems and abstractions, as shown in Table 7.2. For example, the level of abstract mapping of self-understanding was assessed by asking each student to relate (map) two salient abstract self-descriptions to each other, such as attentive and overjoyful in Fig. 7.4. For both support conditions, the responses to the probes were scored based on each skill level (Kennedy, 1994).

Developmental Sequence and Emergence of Levels of Self-Description

The predicted sequence showed high scalability by Guttman scalogram analysis (index of consistency of .96, by Green's [1956] index). The growth patterns under the high support condition showed that the Korean adolescents developed complex self-descriptions comparable to those of

<table>
<thead>
<tr>
<th>Level</th>
<th>Step</th>
<th>Skill</th>
<th>Example of Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rp3:</td>
<td>1</td>
<td>Represenational systems</td>
<td>Person coordinates several concrete self-descriptions together.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Interviewer: What are you like when you are outgoing?</td>
</tr>
<tr>
<td>Ab1:</td>
<td>2</td>
<td>Single abstractions</td>
<td>“By comfortable I mean not feeling awkward or uncomfortable, but being very familiar with and skilled at things. ‘Comfortable’ means having good feelings and not feeling anxious or nervous. A comfortable person is someone who has no worries and believes everything will turn out fine.”</td>
</tr>
<tr>
<td>Ab1:</td>
<td>3</td>
<td>Complex single abstractions</td>
<td>Person shifts between several general abstract attributes or definitions.</td>
</tr>
<tr>
<td>Ab2:</td>
<td>4</td>
<td>Abstract mappings</td>
<td>“By ‘introverted’ I mean that I don’t freely express my inner thoughts and I am not very talkative because I’m shy. I think that I’m plain because I don’t have any good or bad qualities that stand out. I’m wishy-washy and always follow other’s opinions, hiding my own.”</td>
</tr>
<tr>
<td>Ab2:</td>
<td>5</td>
<td>Complex abstract mappings</td>
<td>Person relates or maps three or more abstractions to each other.</td>
</tr>
</tbody>
</table>

(Continued)
TABLE 7.2 (Continued)

<table>
<thead>
<tr>
<th>Step</th>
<th>Skill</th>
<th>Example of Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Person coordinates groups of abstract self-descriptions, integrating them in complex relations with each other.</td>
<td></td>
</tr>
</tbody>
</table>

North American adolescents (Harter & Monsour, 1992; Kennedy, 1994). They developed through three distinct age periods that mapped roughly to the three levels in the abstract tier of skill theory: single abstractions, abstract mappings, and abstract systems. Growth began at the level of single abstractions and spurted with emergence of the second and third levels, as shown in Fig. 7.5.

Moreover, the level of the students' understanding of self-in-relationships varied dramatically as a function of the low and high support conditions of assessment. Figure 7.5 shows that in the low support condition adolescents produced simple descriptions, never rising above single abstractions even at the oldest ages (20 years). On the other hand, in the high support condition students produced complex descriptions comparable to those given by North American adolescents, although the high-level descriptions were delayed by a few years.

The optimal level of self-understanding for the 8th through 10th grades (ages 14 to 17) was characterized by the skill level of single abstractions. Under the high support condition, students incorporated several concrete attributes or behaviors into intangible categories and discussed these at some length. They defined themselves in different relationships as having such abstract personality and emotional characteristics as extroverted (neasongchok), comfortable (pyonan), and free (chayureopa). They could provide complex definitions of these characteristics and shift easily from one abstraction to another. For example, when a 15-year-old boy was asked to explain what he meant by being comfortable (pyonan), he stated: “By comfortable I mean not feeling awkward or uncomfortable, but being very familiar with and skilled at things. ‘Comfortable’ means having good feelings, not feeling anxious or nervous. A comfortable person is someone who has no worries and believes everything will turn out fine.”

At 16 to 17 years of age, the adolescents showed a developmental spurt with the emergence of the optimal level of abstract mappings. By the 11th grade, all the Korean students were functioning at the abstract mapping level under the high support assessment condition, coordinating relations among two or more abstract self-descriptions. They were able to understand how self-defined traits and dispositions were related to each other, as in similarity, opposition, or contradiction. For example, one student saw the characteristics of being comfortable (pyonan) and unsocial (pisagyo-jog) as in opposition: “With my friends I am comfortable and talkative but at school I am the opposite, unsocial, I become speechless, and as a result I become nervous and feel awkward.”

At around age 19 to 20 the Korean adolescents coordinated several abstract self-descriptions into a more coherent self-system, showing a spurt in their optimal level of self-understanding as the level of abstract systems emerged. They systematically related multiple abstract concepts such as two aspects of individuality (kaeinchog)—freedom (chayureun) and self-expression (chagi pyohyop)—to two aspects of society that suppress individuality—“face” or social status maintenance (ch'omyon) and conformity (chunbong). One 20-year-old man said:
To be an individual, you have to have a certain amount of freedom from social conventions in order to express yourself. By this, I mean that one’s thoughts and actions are not affected by outsiders and one can have his own thoughts and carry them out without always conforming to the opinions of others. Freedom means being able to express your opinion and thoughts and carry out actions, while at the same time being responsible for your actions so that you don’t cause harm to others. An individual is a free man whose action is not limited or affected by the environment, nor influenced by pressures to maintain status or reputation in society, but is able to do as he wishes to further develop and express his talents as long as he doesn’t hurt others.

Optimal and Functional Levels of Self-Understanding

The students’ level of self-understanding varied dramatically across assessment conditions. Under the high support condition, students consistently produced higher upper limits of self-understanding than they did under the low support condition, in which they had difficulty expressing themselves.

Functional levels were characterized by references to concrete behaviors and descriptions of simple abstract traits, with only small increases in level with age. The younger students tended to describe themselves in terms of concrete behaviors, “I talk and joke around” or “I tell her (mom) about school.” If they expressed themselves using more abstract traits, they rarely elaborated on them. Even the college freshmen, who under the high support condition demonstrated abstract systems of self-understanding, had difficulty generating more than a complex single abstraction.

In addition, the growth curves for the two conditions were dramatically different, as predicted, with a grade main effect, $F_{5,60} = 18.45, p < .00001$, a condition main effect, $F_{1,60} = 577.61, p < .00001$, and a grade-by-condition interaction, $F_{5,60} = 3.82, p < .0045$. In the low support condition development showed slow monotonic change, whereas in the high support condition it showed discontinuous change, with stagelike spurts in performance, as shown in Fig. 7.5. Age trend analyses found a significant cubic component for age, $F_{3,60} = 35.98, p < .00001$, for the high support condition but not for the low support condition, in which age trends were better described by a linear growth model. There was also a gender effect, $F_{1,60} = 5.69, p < .02$, with girls scoring higher than boys, especially in the high support condition.

In summary, there was a clear developmental sequence for self-understanding among Korean adolescents, and the main predictions about growth functions were confirmed: Under high support conditions, the adolescents showed high levels of understanding of self-in-relationships, contrary to previous findings of simple self-understanding in Asian adolescents and young adults. Moreover, this understanding demonstrated two growth spurts, first with the emergence of abstract mappings between tenth and eleventh grades, and second with the emergence of abstract systems between 12th and 13th grades. The ages of these spurts, especially the first one, were delayed by 1 or 2 years relative to American adolescents (Harter & Monsour, 1992; Kennedy, 1991; 1994).

In contrast, under low support conditions Korean adolescents showed extremely low levels of self-understanding, as well as only minimal increases in level across the 6-year span of the sample. This result replicates prior research assessing self-understanding in Asian cultures, which found low levels of self-understanding. The earlier studies used assessment procedures that provided little support for high-level functioning, although the investigators did not recognize that their procedures were producing a low estimate of Asian students’ levels of self-understanding.

7. TOOLS FOR ANALYZING THE MANY SHAPES OF DEVELOPMENT

The research assessing adolescents’ understanding of self-in-relationships provides one important way of testing the dynamic skills analysis of developmental processes, but growth modeling greatly increases the power of this test by determining whether the processes specified do in fact produce the growth curves obtained in the study. Do the processes specified in the dynamic skills analysis in fact produce growth like that shown in Fig. 7.5? For example, do optimal levels show stage-like spurts to highly complex self-understanding while functional levels undergo monotonic growth to simpler self-understanding?

Building a specific growth model embodying the hypothesized skill processes provides a powerful test of the theory. Key components in building a model are the nature of the basic growth process for each individual grower, the kinds of hierarchical connections among growers in a domain, and the kinds of weaker connections of growers within and across domains. In the model, each grower is one of the behaviors being assessed in a domain at a level, and the distinct relationships (mother, father, best friend, etc.) are the domains. The hierarchical connections among growers are the ways that later growers depend on earlier ones to be able to grow. The connections among growers across domains are the influences that growers in one role relationship have on growers in another role relationship. The connections among growers in the model illustrate several important types of relations and the large effects they can have on the shapes of growth curves.
Logistic Growth and Nonlinear Change

The basic growth function for each grower is logistic, with the level of a behavior on one trial (or event) depending on its level at the previous trial plus its rate of growth and its carrying capacity. Note that the term level in growth equations refers to some quantity that a grower has reached. That quantity could be any of several different characteristics, such as frequency of response or amount of time spent in an activity as well as level in the sense of skill theory. The meaning of level will depend upon how the growth processes are defined. In the current growth models, level \( L \) in the growth equations refers to the complexity of a behavior along a scale defined by skill theory. In the simple case, the unit intervals mark movement to the next level of skill complexity, but often they refer to some arbitrary number of steps along a developmental scale.

The equation produces mostly S-shaped growth when it operates alone, without connections to other growers, as shown by Growers 1 and 2 in Fig. 7.6. Even for this unconnected growth equation, however, there is important variation in the growth curve depending on the values of the three parameters, as illustrated by the turbulence in Grower 3 as it nears its carrying capacity.

There are several forms of the logistic growth equation. The form of logistic growth in Fig. 7.6, which is the model we assume for development of self-in-relationships, is specified by the following equation for Grower \( B \):

\[
L_{B_{t+1}} = L_B + \frac{L_B^2}{K_B} - R_B \frac{L_B}{K_B}
\]  

(1)

\( L_{B_{t+1}} \) is the level of Grower \( B \), with subscript \( t \) indicating the previous trial and \( t + 1 \) indicating the current trial.

\( R_B \) is the rate of growth of Grower \( B \), designating the amount of change in each trial.

\( K_B \) is the carrying capacity of Grower \( B \), which is the limit on growth for this particular system.

That is, the level in the current trial derives from the combination of three terms. First is the level in the previous trial, on which the current trial builds. Second is the growth term—the growth rate times the square of the level in the previous trial divided by the square of the carrying capacity. In simple growth curves, this factor leads to increase on each trial, with the increase determined by both the growth rate of the system, the level on the previous trial, and the carrying capacity of the system. Dividing

by the carrying capacity treats the level as a ratio of the system's capacity instead of as an absolute value. Such relative values reflect the assumption that the system's level operates in terms of the capacity of the system, and they also produce more stable systems than absolute values.

The growth term in this form of the logistic equation squares the ratio of level to carrying capacity. In a simpler form of the equation, the growth term uses the ratio without squaring. The rationale for squaring the term is that psychological growth seems to depend doubly on the person's prior level—once because current understanding of self-in-relationships is built on earlier understanding and a second time because the level affects the likelihood that the person will encounter situations that promote growth. Van Geert (1994) elaborated this argument and also showed that this form of the growth equation fits some actual growth curves better than the simpler equation (Ruhland & van Geert, 1996).

The third term provides a form of regulation, correcting for growth based on the limits of the system. It subtracts an amount that keeps the system from exploding beyond its carrying capacity. The amount subtracted is the product of the growth term multiplied by the ratio of the level to the carrying capacity, which makes the level and carrying capacity cubed instead of squared. When the current level is low relative to the carrying capacity, little is subtracted, but when the current level becomes high, a much larger amount is subtracted. As the level approaches the carrying capacity, this term becomes large enough to
cancel out most growth. As a result the system approaches the carrying capacity as a limit, at least for simple cases. Grower 3 in Fig. 7.6 shows how when the growth rate is high, this limit can lead to turbulent fluctuations in growth as the level approaches the carrying capacity.

**Hierarchical Connections**

In a strand or domain in the developmental web, each successive step builds on the previous step hierarchically, as when people’s conception of themselves as being secure with their mothers is built on earlier concrete representations of activities in which their mothers provided affection or help when it was needed. In the growth model, each strand is composed of a series of connected growers built successively upon each other. Each grower changes as a result of a logistic growth equation like that in Equation 1, but growth at later steps begins only after skills at an earlier step are sufficiently strong and frequent for people to build on. For example, adolescents can begin to construct an abstract concept of security with mother only when they have sufficient understanding that their mother will nurture them in several specific situations. In one simple case, growth in a hierarchy reflects a succession of logistic growers building on each other and thus shows a series of S-shaped growth curves, like that for optimal level in Fig. 7.5.

In such a prerequisite connection, a later grower does not begin to grow until the prior, prerequisite grower has reached some minimal level. This relation is represented straightforwardly in the growth equation as a prerequisite determining when a later grower along a strand can begin to grow. In a five-level hierarchy, Grower A is a prerequisite or precursor for Grower B, B for C, C for D, and D for E. Grower A, the first one in the hierarchy, has no prerequisite specified.

\[
L_{B_{t+1}} = L_B + P_{B} \left[ \frac{L_B}{K_B} - \frac{L_{A}}{K_{A}} \right]
\]

(2)

\[
P_B = \begin{cases} 
0 & \text{if } L_A < p \\
1 & \text{if } L_A \geq p
\end{cases}
\]

(3)

When the prerequisite Grower A has not yet reached some specified level \(p\) at time \(t\), such as .3, then precursor \(P_B\) is 0, and Grower B does not grow. When Grower A reaches .3, precursor \(P_B\) becomes 1 and Grower B begins to grow. Of course, the specification of when the precursor is reached can be more complex than simply one trial at .3. For example, Grower A might need to reach .3 for some number of trials before Grower B can begin to grow.

In addition to the strong precursor connection, the model contains two other kinds of weaker hierarchical connection: support and competition between successive growers in the hierarchy. As defined by skill processes, a higher level skill such as Grower B potentially supports and competes with the skill that it is built upon from the prior level, Grower A.

Support of Grower A by Grower B is the product of the strength of support, times the level of Grower B divided by the carrying capacity of B. This term is added to the growth equation for Grower A.

\[
+Sh_{B \rightarrow A} \frac{L_B}{K_B}
\]

(4)

\(Sh_{B \rightarrow A}\) is the parameter specifying the strength of the supportive effect of Grower B on Grower A.

For example, when a girl constructs an abstraction for maternal nurturance, her understanding that her mother has nurtured her in several ways can support her understanding of her mother’s concrete nurturing activities, such as specific ways that her mother helps her with difficulties at school. This kind of support from above turns out to be an important contributor to developmental spurts in growth curves, promoting the occurrence of growth patterns like the succession of spurts seen in Fig. 7.1. It thus helps to explain empirical findings of successive spurts in growth curves like that for the Korean study (Fig. 7.5). As a result of this support, the spurts can move a grower’s level above its initial carrying capacity, producing much higher levels and boosting its effective carrying capacity.

The competitive effect of Grower B on Grower A is the product of the strength of competition times the change in the level of Grower B on the two prior trials divided by the level of B on the prior trial. This term is subtracted from the growth equation for Grower A.

\[
-L_{B_{t-1}} - \frac{C_{B \rightarrow A} L_{B_{t-1}}}{L_B}
\]

(5)

\(C_{B \rightarrow A}\) is the parameter specifying the strength of the competitive effect of Grower B on Grower A.

The competition is based on the change in the level of Grower B rather than the level itself because the effort involved in producing growth is posited as the source of competition. For example, when an adolescent is working to construct an abstraction for the nurturance of her mother, the time and effort she spends building that concept cannot be used to improve some component from the prior level, such as her understanding
how her mother supports her in dealing with difficulties at school. This use of time and energy is how Grower B competes with Grower A, not through the level of skill itself.

When the support and competition processes are added to the growth model in equation 2, then the equation becomes:

\[
L_{A_{it}} = L_{A_i} + P_{A_i} \left[ R_{A_i} K_{A_i} - R_{A_i} K_{A_i} \right] \\
+ S_{B_i \rightarrow A_i} K_{B_i} - C_{B_i \rightarrow A_i} \left( \frac{L_{A_i} - L_{A_{i-1}}}{L_{A_i}} \right)
\]

(6)

Each successive level in the hierarchy involves a similar growth equation, such that Grower C supports and competes with Grower B, Grower D supports and competes with Grower C, and Grower E supports and competes with Grower D. The model for the self-in-relationships study contains five successive levels, A through E. In addition, each level includes five domains that are connected with each other.

**Connections Between Domains**

Between domains (strands) there are also supportive and competitive connections. The model contains five domains, each representing roles for defining self-in-relationships, such as mother, father, best friend, romantic friend, and teacher. The growers for domains at Level A are A1 through A5, for Level B they are B1 through B5, and so forth. To capture the gist of multiple connections, we connect five paired domains within each level, with one member of the pair supporting and competing with the other: 2 affects 1, 3 affects 2, 4 affects 3, 5 affects 4, and 1 affects 5.

Support of Grower A2 by Grower A1 is the product of the strength of support times the level of Grower A2 divided by a term that approaches zero as level approaches carrying capacity (one minus the level of A2 divided by its carrying capacity). The rationale for this limiting term is that when a skill in a domain is first being constructed, its effect on related domains is more powerful because it involves new capabilities and understandings. This support effect is added to the growth equation for Grower A1.

\[
+ S_{A2 \rightarrow A1} L_{A1} \left( 1 - \frac{L_{A2}}{K_{A2}} \right)
\]

(7)

\( S_{A2 \rightarrow A1} \) is the parameter specifying the strength of the supportive effect of Grower A2 on Grower A1.

For example, a girl’s construction of an abstract concept for maternal nurturance can support growth of a concept for her feeling of security with her mother. This kind of support contributes to the specific shape of developmental spurts in growth curves. For some values, it pushes growth spurts above carrying capacity, with skill level gradually returning to the carrying capacity limit. That is, support between tasks within a domain can produce an overshoot followed by a drop in level, producing a period of U-shaped growth, as seen in the upside-down U’s at the top of spurts in Fig. 7.1 and in the findings from the Korean study for growth between Grades 11 and 13 in Fig. 7.5. This kind of U-shaped growth has been found commonly in hierarchical growth data (Bever, 1982; Fischer & Rose, 1994; Strauss with Stavy, 1982).

The competitive effect of Grower A2 on Grower A1 is the product of the strength of competition times the change in the level of Grower A2 on the two prior trials divided by the level of A2 on the prior trial. This competition effect is subtracted from the growth equation for Grower A.

\[
-C_{A2 \rightarrow A1} \frac{L_{A2} - L_{A_{2-1}}}{L_{A_{2-1}}}
\]

(8)

\( C_{A2 \rightarrow A1} \) is the parameter specifying the strength of the competitive effect of Grower A2 on A1.

Like competition between levels, this competition between domains within a level is based on the change in the level of Grower A2 relative to its prior level. For example, an adolescent’s construction of an abstract concept for the nurturance of her mother takes time and energy that cannot be used to construct a concept for her mother’s inhibition in talking with her about feelings.

Support and competition within levels are added to equation 6 to make the complete equation for a grower in the self-in-relationships model:

\[
L_{A_{i+1}} = L_{A_i} + P_{A_i} \left[ R_{A_i} K_{A_i} - R_{A_i} K_{A_i} \right] \\
+ S_{B_i \rightarrow A_i} K_{B_i} - C_{B_i \rightarrow A_i} \left( \frac{L_{A_i} - L_{A_{i-1}}}{L_{A_i}} \right) \\
+ S_{A2 \rightarrow A1} L_{A1} \left( 1 - \frac{L_{A2}}{K_{A2}} \right) - C_{A2 \rightarrow A1} \frac{L_{A2} - L_{A_{2-1}}}{L_{A_{2-1}}}
\]

(9)

Every one of the 25 growers in the model has a comparable equation, except for two variations. The growers at the highest level have no between-level
connections because they have no level above them, and the growers at the lowest level have no precursor because they have no level below them.

Other Possible Variables

In these equations, rate \( R \) and carrying capacity \( K \) are constants for each grower, remaining the same on every trial. They can also be variables instead of constants, depending upon the conception of growth processes being modeled. In the model for self-in-relationships, rate is a constant, but carrying capacity is a variable. The effects of connections often alter a grower’s effective carrying capacity, as when within-level support moves level above initial carrying capacity in the U-shaped portions of Grower \( y \) in Fig. 7.1. Support relations are especially likely to alter effective carrying capacity because they depend on a grower’s level rather than its change in Fig. 7.1. Support relations are especially likely to alter effective carrying capacity because they depend on a grower’s level rather than its change from trial to trial. To reflect the alterations in effective carrying capacity, the model determines carrying capacity on each trial by using the effective carrying capacity for that grower on the previous trial. The effective carrying capacity is the sum of the initial carrying capacity plus the changes in carrying capacity caused by the effects of connections. (The changes must be divided by rate to place them on the same scale as the initial carrying capacity.)

\[
K_{AI,t} = K_{AI} + \left( SB_{AI} - \frac{L_{BA} - L_{AI,t-1}}{L_{AI}} \right) + SW_{AI} \left( 1 - \frac{L_{A2}}{K_{AI}} \right) - Cw_{AI} \left( 1 - \frac{L_{A2}}{L_{AI}} \right)/R_{AI}
\]  

(10)

These new values of \( K_{AI,t} \) are substituted in the earlier equations for each trial. For a wide range of parameter values, this refinement of carrying capacity has only small effects, but for some sets of values it changes the shapes of growth.

To produce a score for a hierarchical developmental scale including all five levels in a domain, the growth values for the grower at each level are summed. The resulting score is similar to the Guttman scales that are used frequently in research on hierarchical cognitive development, in which each successive step in a sequence is scored with the next higher number on the scale.

**SHAPES OF DEVELOPMENT IN THE HIERARCHICAL MODEL**

In the model of growth of self-in-relationships, skills in five domains grow through five hierarchical levels, changing dynamically through a wide array of shapes of development. For this model, the domains are representations of self in several important relationships, such as mother, father, best friend, sibling, and teacher. The five levels are representational systems, single abstractions, abstract mappings, abstract systems, and principles integrating abstract systems (Fischer, Kenny, & Pipp, 1990; Kennedy, 1994). (Only the first four levels were actually measured in the study, but the fifth one needs to be included to capture the appropriate range of variation in growth curves. Children also develop through several levels prior to the first one in the model, but those earlier levels were not the focus of the Korean study.)

To test the dynamic skills framework for development of self-in-relationships, we compared the growth curves obtained in the self-in-relationships study for optimal and functional levels with those produced by the model. The model can be used to investigate concepts and hypotheses about developmental processes beyond the data of the study, including not only optimal versus functional levels and U-shaped growth but also equilibration, turbulence, and catastrophe.

**Optimal and Functional Levels**

The findings for development of self-in-relationships in Korea include a dramatic difference between optimal and functional levels. Under optimal conditions, adolescents showed relatively rapid growth as well as two successive spurts in understanding, as shown in Fig. 7.5. Under functional conditions, they underwent relatively slow growth showing monotonic increase.

The growth model produces clear optimal- and functional-level growth patterns, as shown in Fig. 7.7. With a high growth rate development shows successive spurts that are similar across domains. With low rate, on the other hand, development shows mostly smooth, monotonic increase, with occasional unsystematic irregularities that average out across domains. The domains in Fig. 7.7 start with small differences in initial level across domains and otherwise have identical growth-parameter values except for rate. This variation from growth through a series of spurts to monotonic growth defines a broad set of the growth patterns for the model, as illustrated also by the growth curves in Fig. 7.1 (Growers \( x, y, \) and \( z \)), which derive from the same model.

An important next step in refinement of these tools is to devise methods of testing data directly against the growth model. We are attempting to build tools for using structural equation modeling to estimate the values of the parameters in the model and then test the fit of the data to the model. Although structural equation modeling is designed for linear variation and our model is nonlinear, what is nonlinear is the outcomes of the growth models, not the parameters. The parameters are primarily
Equilibration, Regulation, and the Piaget Effect

Piaget (1957; 1967/1971; 1975) characterized the process of development as *equilibration*, in which multiple influences are regulated by a person's activity to produce a series of successive equilibria, which are built on each other hierarchically. Spurt-and-plateau growth patterns are one form of equilibration, although Piaget did not explicitly focus on them. In addition to spurts and plateaus, equilibration involves the quelling of perturbations: Factors that produce jumps or drops in growth should be dampened out so that the system returns to equilibrium levels. In nonlinear dynamics these equilibria are often referred to as attractors rather than equilibria, because the system seeks equilibrium for only some values of parameters (Thelen & Smith, 1994; van Geert, 1994).

The hierarchical growth model for self-in-relationships produces exactly this kind of growth pattern. Figure 7.7 shows growth through a series of optimal equilibria for five distinct domains, with perturbations returning to equilibrium levels repeatedly over time. By the time the domains reach their final equilibria, they are all at the same value—a single equilibrium. Domain 2 illustrates the equilibration especially dramatically. The initial levels \( L \) for Domain 2 were set consistently above those for the other domains (a mean of .065 instead of .058), and these starting levels led to markedly higher spurts with the emergence of each new optimal level. Nevertheless, Domain 2 showed equilibration, dropping substantially toward the other domains and eventually matching their equilibria.

The regulation of higher levels downward toward equilibrium frequently involves U-shaped growth or "regression," as is evident especially for Domain 2 in Figs. 7.7 and 7.8. In the history of child development, scholars have often puzzled about the nature and existence of U-shaped growth (Bever, 1962; Strauss with Stavy, 1962). In our research the peaks of optimal-level growth have frequently been followed by drops (Fischer et al., 1984; Fischer & Rose, 1994). In tests of the dynamic skill model, the most important parameter in producing this U-shaped growth has turned out to be support among domains within a level: Growers in multiple domains reinforce each other as they grow, with the result that they overshoot their natural equilibrium level. Then gradually the system regulates their growth down to the equilibrium point.

Growth does not always show such orderly equilibration, however, as illustrated by the two functional-level growers in Fig. 7.7. (Only two are shown to keep the graph readable.) At times, the model produces dispersion of growers rather than common equilibration. A particularly striking example of this dispersion involves a phenomenon that Piaget (1935/1970;
1947/1950) described when he criticized efforts to accelerate early development. He said that pushing children beyond their natural levels was similar to training animals to do tricks to perform in a circus: It did not contribute to their normal growth and could lead to stunted development in the long run. Figure 7.8 shows this phenomenon with the hierarchical growth model, and we have named it the Piaget effect in his honor. Values for parameters are identical to those for Fig. 7.7 except that the initial levels for Domain 2 have the same mean as other domains. A single change has been then introduced—a one-time boost to the rate of development, analogous to special training to produce precocity. After this boost, the rate parameter returns to the normal levels shared with all other domains. (Similar effects result from several ways of raising the rate, including increasing directly the parameter for rate $R$, or increasing the parameter for within-level support at Level B for Domain 2, $Sw_{B1}$).

The boost in rate increases Domain 2 well above other domains during the emergence of the second level, but the pattern changes dramatically as growth proceeds: For later growth Domain 2 shows much lower levels of skill, as shown in Fig. 7.8. In addition, the five domains no longer show common or similar equilibria but instead spread out across different equilibria. The short-term change in one grower thus has a pervasive effect, throwing the entire system out of equilibrium and producing a lower eventual level in the domain that receives an early boost. As Piaget suggested, an early boost to growth produces stunted development in the long run (for some parameter values).

**Turbulence and Catastrophe**

The self-in-relationships model is a complex nonlinear system, and like most such systems it produces widely different growth patterns. The Piaget effect is only one example of the many variations in growth. For many values of the parameters, the growth process simply explodes or collapses, as turbulence in one place reverberates through the connections among growers and disrupts orderly growth. Besides explosion of the system to impossibly high levels or collapse to zero or negative values, growth often shows turbulent variation or oscillation, as illustrated in a modest form in Fig. 7.1 (Grower $y$). At times the turbulence in the system has the specific properties of chaos as defined mathematically (van Geert, 1994).

High amounts of within-level support between domains often produce such turbulence or oscillation. This higher support creates disruptive feedback between growers and thus can produce turbulence, explosion, or collapse across domains. These nonequilibrium patterns are especially likely when growth rates or feedback processes are too high. More modest rates of growth or feedback usually produce stable growth, sometimes nearly monotonic and sometimes moving through a series of fits and starts.

Interestingly, measures of brain electrical activity (electroencephalogram or EEG) commonly show oscillations in their growth curves that are chaotic, and growth curves for the EEG show dynamic growth processes. Growth of measures of connection among cortical regions based on cross-correlations of EEG wave patterns, called coherence, typically show regular oscillations as they grow (Thatcher, 1994). Growth of the energy in the electrical activity itself shows patterns that are often similar to the curves for the self-in-relationships hierarchical growth model, as in Figs. 7.5 and 7.7 (Fischer & Rose, 1994).

As promising as the self-in-relationships model has proven to be, there are clearly additional steps that need to be taken to broaden and strengthen the tools we have described for developmental analysis. Future needs include applying the tools to a range of other types of growth, such as oscillatory growth and changes in behaviors that are not hierarchical. Another promising direction is integrating neural network models of coordination (Grossberg, 1988; Shultz, Schmidt, Buckingham, & Mareschal, 1995) into the current model to reflect more directly the kinds of learning that occur as children build hierarchical skills.

**CONCLUSION: PROPERTIES OF CONNECTED HIERARCHICAL GROWTH**

The tools of dynamic skill modeling not only help to describe and explain the development of self-in-relationships in Korean adolescents, but they also present a rich portrait of processes of hierarchical psychological growth.

Hierarchical growth involves skills or activities being constructed upon earlier skills or activities. Virtually all theoretical descriptions of hierarchical growth characterize later levels as prerequisites for earlier ones: Lower growers can be constructed only when earlier ones have reached some minimal level of functioning. However, many other types of connections contribute to hierarchical growth, and they tend to be neglected in other developmental models, even in neural network models of growth and development.

Hierarchical growers affect each other in several ways besides the prerequisite relation. They can support each other, as when growers later in the hierarchy support change in growers earlier in the hierarchy—the reverse of the prerequisite process. They can also compete with each other, as when the construction of growers later in the hierarchy interferes with earlier growers. These processes of support and competition contribute substantially to hierarchical growth, often producing nonlinear growth patterns, such as plateaus and plateaus, that go beyond any prerequisite relation.
In addition, hierarchical tasks are affected by other tasks in connected domains, as psychometric researchers have long acknowledged. The domains are primarily independent or fractionated, and therefore the effects are often small, seldom accounting for more than 5% of the variance between two tasks measured at the same time. Nevertheless, they can have powerful long-term consequences when the effects of connection accumulate across time or across tasks.

Two common kinds of connections between domains are support and competition. Although the concepts of support and competition are similar to those for connections between levels, these processes can occur for tasks that are located entirely within a level, and their effects can be important. For example, spurts upon emergence of a new optimal level are often magnified into overshoots by connections between domains within a level. U-shaped growth following spurts is often a consequence of the effects of within-domain support, as the overshoots gradually return to the natural levels of the system (the points of equilibrium or attraction).

Hierarchical growth models produce a wide array of different growth patterns. They sometimes show virtually monotonic growth. When growth rates are relatively high, the range of competence increases substantially, and growers commonly move through a series of spurts and plateaus (or drops), like a sequence of S-shaped logistic curves connected to each other. This variation explains the developmental range found in a number of developmental studies, including the study of development of self-in-relationships in Korea. Contextually supported assessment produces high levels of competence and stage-like spurts, whereas spontaneous, unsupported assessment produces low levels of competence that grows monotonically. When growth rates or connection processes are too strong, growth explodes, collapses, or varies in a turbulent fashion. For growth to be equilibrated or monotonic, growth must be relatively slow and steady, and influences across domains must be moderate.

These tools for dynamic analysis provide powerful methods of characterizing growth and development precisely and explicating change processes far beyond the global verbal treatments of most developmental theory. In this chapter we have outlined one way of using the tools for precise analysis of hierarchical cognitive development. Use of the tools of dynamic growth analysis in this way can help immeasurably to strengthen the explanation and prediction of human growth and development.

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