Formulating Measures:

toward modeling in the K-12 science and mathematics curriculum

Humans seem to have an insatiable desire to rank order. Consider the ubiquitous "10 most…" and "10 best…" lists that are found in popular magazines. The annual rank ordering by news weeklies of US colleges and universities is anxiously awaited each year by admissions offices throughout the country.

Sometimes we seek to order quantities that are directly accessible to the senses such as heights and weights. The rank ordering of such quantities is relatively straightforward, although it may require the introduction of instruments to refine or extend our senses. Sometimes, however, we seek to order quantities that are clearly dependent on several, or even many, contributing factors such as cost of living or the efficiency of an automobile.

It is sometimes the case that when rank ordering such complex quantities, the bases for the rank orderings are made explicit and public. This may permit one to say that in the ordering A, B, C whether B is closer to A than it is to C and if so, by how much. When this happens it is fair to say that we have devised a measure of the scale along which A, B, and C are arrayed.

It is centrally important to the practice of many natural and social science disciplines to go through the exercise of forming measures[1] of quantities of interest. Indeed it may be said that the formulation of measures is an important first step in the act of modeling – with the measure itself serving as the heart of a structural model, or the time dependence of the measure as the core of a functional model[2].

Geographers[3] studying spatial segregation patterns, computer scientists[4] studying the evolution of biological complexity, biochemists[5] trying to establish similarity of metabolic pathways, reading specialists[6] studying dyslexia in children and public health physicians[7] studying stress in neighborhoods to name a few.

---

[1] By *measure* I mean a construct that we devise to quantify the amount or degree of a property or attribute of an object or a situation of interest.

[2] See for example – Schwartz, J.L., in Foundations for the Future in Mathematics Education, R.A. Lesh, E. Hamilton & J.J. Kaput, (Eds.), Mahwah, NJ 2007, Lawrence Erlbaum Associates, pp. 161-172

[3] See for example David W. Wong, Formulating a General Spatial Segregation Measure, The Professional Geographer, Volume 57 Issue 2 Page 285 - May 2005

[4] In a report to the NSF from the Digital Life Laboratory at CalTech - The importance of formulating a measure of complexity for symbolic sequences cannot be underestimated, as discussions about complex systems have always been marred by an inability among scientists to agree on a measure. The practical success of this measure, as well as its intuitive simplicity, should ultimately remedy this situation. We have applied it to one of the central problems in evolutionary theory: the evolution of complexity

[5] See for example Maureen Heymans and Ambuj K. Singh, Deriving phylogenetic trees from the similarityanalysis of metabolic pathways, Bioinformatics,Vol. 19, Suppl. 1, 2003 pp, i138-i146

[6] See for example - http://web.mit.edu/murj/www/v07/v07-News/v07-world.pdf.

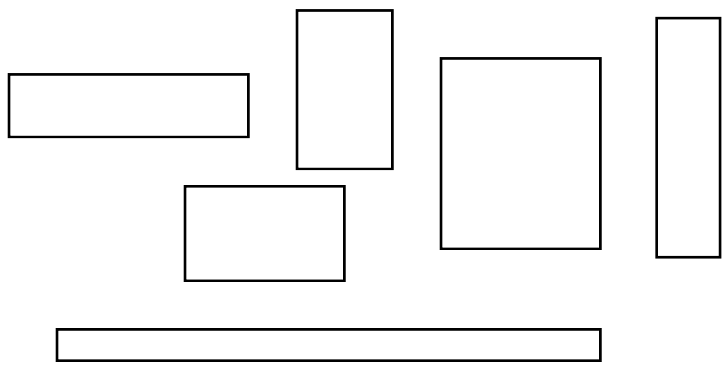
[7] See for example – A. Steptoe & P.J. Feldman, Neighborhood Problems as Sources of Chronic Stress: Development of a Measure of Neighborhood Problems, and Associations With Socioeconomic Status and Health, Annals of Behavioral Medicine 2001, Vol. 23, No. 3, Pages 177-185

Formulating a measure requires one to be explicit about the constituent elements of data that one believes are important in a given situation. In this respect, formulating a measure is like making a model – one must decide on the basis of relevance and importance, what to include in, and exclude from, the measure being formulated.

As an example of such a measure-formulating activity, let me offer an example from the work of the Balanced Assessment in Mathematics[8] project. The project devised a set of tasks called "-ness" tasks (for reasons that will presently become clear). The purpose of these tasks is to see how well students can formally describe a relationship of which they are aware perceptually but probably have never attempted to describe in any formal, not to mention quantitative, fashion. For the sake of specificity, the following is an example:

"Square-Ness"

Below is a collection of rectangles.



1. Which of the rectangles is the "squarest"?
2. Arrange the rectangles in order of "square-ness" from most to least square.
3. Devise a measure of "square-ness," expressed algebraically, that allows you to order any collection of rectangles in order of "squareness."
4. Devise a second measure of "square-ness" and discuss the advantages and disadvantages of each of your measures.

These tasks require students to identify and describe formally a geometric property of some two- or three-dimensional shape. It is important to stress that properties such as "squareness" are not formal geometric properties. There are no academically correct or universally accepted answers to these questions. On the other hand, there are sensible (and non-sensible) answers.

Elements of performance on this task include:

- Choosing the most and least square figure
- A verbal description of the geometric property in question (in this case what one means by "squareness")

[8] See balancedassessment.concord.org

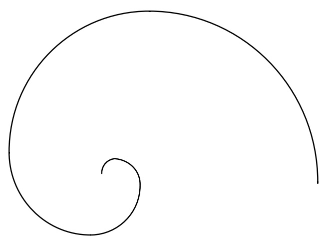
- Identifying the geometric elements that combine to form the measure of “squareness”
- Forming an algebraic or other symbolic relationship among these elements
- Computing values of the measure for various figures
- Discussing the advantages and disadvantages of the measure under varying circumstances

This task can clearly be extended to include defining measure of “squareness” for all parallelograms, or for all quadrilaterals, or for that matter for all closed convex curves in the plane.

Other such tasks involve defining measures of “sharp-ness” of bends in roads[9] (not as obvious as one might think), “crowded-ness”[10], “smooth-ness” of “spherical” objects[11]

---

[9] For example - Here is a map of a part of a trail in a park.



Pretend you are talking to a friend on the telephone. Use your measure of “curvy-ness” to tell your friend how to draw this map.

[10] Which is more crowded? How much more crowded?

[11] Pretty smooth! smoother than an orange?

(e.g., golf ball, orange, Earth, etc.), “disc-ness”[12] of cylinders (e.g., tuna fish can, penny, stick of uncooked spaghetti, etc.), “good-ness” of fit of a functional form to a collection of data points, etc.

The essential point in such tasks is this – by asking people to formulate measures and thus to formally isolate elements of a perceptually familiar image and to describe with precision the relationships among them, the task focuses on an act of modeling largely unconfounded by the need for specialized domain knowledge.

Square-ness and the other –ness tasks discussed above are instances of measure formulation that proceed from the perceptually salient. They are elementary forms of modeling tasks that unfortunately are too rarely found in the K-12 curriculum despite the fact that even very young children can make substantial headway with them.

Measure-formulation can and should go beyond the kinds of examples shown above that start with stimuli that can be perceived visually. Indeed, there are two instances of K-12 measure formulation that are present in all K-12 mathematics and science curricula although they are almost never presented as such. They are

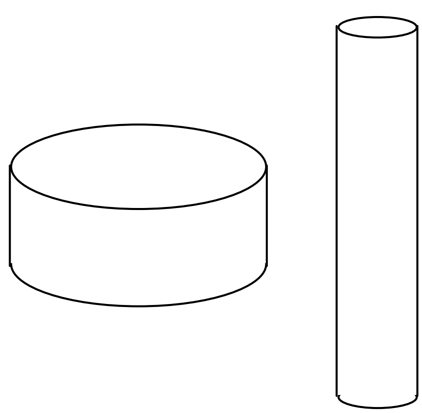
- *Quotient measures* – for the most part these are intensive quantities such as ratio and rate [mi/hr, mi/gal, hr/mi, gal/mi, $/lb, lb/$, …] ,

and

- *Product measures* of all sorts but especially Cartesian products such as areas, volumes, kinds of outfits, passenger-miles, person-years, etc.

---

[12] A coin is a disc, and an uncooked piece of spaghetti is a cylinder. If you think about it, however, a coin is also a cylinder and an uncooked piece of spaghetti is also a disc. Clearly the coin is more disc-like and the spaghetti more cylinder-like.



Given a coin, a tuna-fish can and a soup can: Devise a measure of disc-ness that allows you to say which object is the most disc-like and which is the least and whether the remaining object is more like the most- or least- disc-like. By how much?

An intensive quantity is a measure of a relationship and *not* of the amounts of the quantities that enter into that relationship. If we wish to capture in the measure we formulate a relationship that does not depend on the amounts of the quantities that enter into the relationship we often use an intensive quantity. For example, in the case of square-ness illustrated above, [denoting length of horizontal sides by H and vertical sides by V] the measure H/V will be the same for all geometrically similar rectangles independent of size.[13]

The density of a material is a measure of the relationship between the mass of an object made of that material and the volume the object occupies. If we were to shatter the object[14] into many pieces, the pieces would each have smaller masses and smaller volumes, but they would all have the same density. Density is a measure that characterizes the nature of a material but *not* its amount. Formulating measures like this is a notoriously difficult task – if for no other reason than we both must decide on what properties to attend to, and then to formulate some combination of these properties whose value will not depend on how large a piece of the material we are considering.

Here is an excerpt from an essay that follows the reasoning leading to the formulation of a density measure.

> ***If we have a collection of pieces of aluminum (say, an aluminum saucepan, a door hinge, a machine screw) and a collection of pieces of pine (say, a cutting board, a small jewelry box, a chess piece) and a collection of pieces of polystyrene (say, several different food containers) most people would be willing to agree that there is some property of aluminum that is "larger" than that of pine. Suppose, for the moment, we call this property "oomph"***
>
> ***Is there some way we can pin down what such a property called "oomph" might be? More to the point, is there some way we can say whether the "oomph" of some third material like polystyrene is closer to the "oomph" of aluminum than it is to the "oomph" of pine? And if so, how much closer?***
>
> ***More generally, if we have a bunch of different materials, can we find a way to order them according to the "oomph" property – from the one with the "largest" to the one with the "smallest"?***
>
> ***First thoughts would suggest that the relevant property of the aluminum pieces, the pine pieces and the polystyrene pieces that we want to pay attention to is how "big" they are. But by "big" we might mean their physical size (volume – amount of space they take up) or we might mean their weight (or their mass). Clearly each piece of aluminum, pine and polystyrene has <u>both</u> a volume and a mass.***
>
> ***Assuming we have the proper instruments for measuring volume and mass, we could measure both the volume and the mass for each of the pieces of aluminum, pine and polystyrene. If we do so, we will quickly discover that our mystery property, "oomph", cannot be mass and it cannot be volume. We can find some pieces of pine with more***

---

[13] This may be regarded by many as a poor measure since rotating the rectangle by 90 degrees causes it measure of square-ness to change. A better intensive measure might be [(H-V)/(H+V)]. On rotation by 90 degrees the sign, but not the absolute value, of the squareness changes.

[14] …assuming the object is made of some homogeneous material

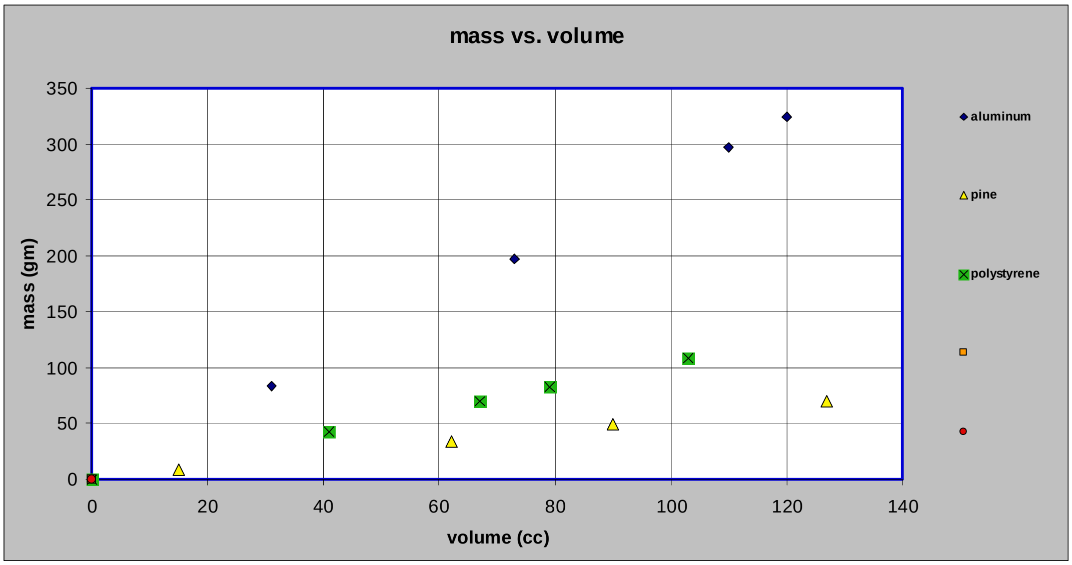
mass than pieces of aluminum and some pieces of pine with less mass than pieces of aluminum.

If we try to organize our data we might make a table something like this:

material 1 volume (cc)	aluminum mass (gm)		material 2 volume (cc)	pine mass (gm)		material 3 volume (cc)	polystyrene mass (gm)
0	0		0	0		0	0
120	324		15	8.3		41	43.1
73	197		62	34.1		103	108.2
31	83.7		127	69.9		79	83
110	297		90	49.5		67	70.4

[*Note that for each material we have included the data point volume = 0 cc corresponding to mass = 0 gm. This is because if an object has no mass it takes up no space.*]

Or we might plot our points on graph paper something like this:



<u>Data in tables (symbols) and graphs (images)</u>

It is important to stress that at this point all we have done is to record our <u>data</u>. We have used two different but equivalent representations, one involving symbols (tables) and the other involving images (graphs). Neither representation is more fundamental than the other. Each representation has its advantages. The table records our data in a way that makes the precision of our measured numbers clear. However, it does not make salient whether the masses or the volumes are close to one another or not. The graph, on the other hand, typically does not allow us to read the values of our measurements to the same degree of precision as the table. It does, however, show us whether data points are close to or far apart from one another and it does suggest patterns and regularities that may be present in the table but not apparent.

Both the tabular and the graphical representations of the mass and volume data we have collected point to the formulation of an intensive quantity as a quotient measure of the relationship between mass and volume for a given material. The tabular representation suggests for any given material the relationship mass = [constant of proportionality] x volume

holds. The [constant of proportionality] must have the dimensions of mass/volume if the relationship is to be dimensionally meaningful. The [constant of proportionality] of a given material is called the density of the material. Thus, having characterized the relationship by a single number (albeit with units) we are in a position to rank order these numbers.

The graphical representation makes the possibility of rank ordering the relationship between mass and volume even more salient that does the tabular representation. The data points for any given material lie on a straight line going through the origin and we can rank order these lines by their slopes.

Another kind of measure found in the K-12 mathematics and science curricula is product measure, typically a Cartesian product.

Some examples include

- area [~length•length]
- volume [~length•length•length]
- combinations[~counts•counts]
- work [~force•distance]
- momentum [~mass•velocity]
- flux [~(mass/distance)•(distance/time){1D}, ~(mass/volume)•(volume/time){3D}]

etc.

Cartesian products seem to be a natural structure to turn to when one is confronted with a quantity that seems to depend on two constituent quantities such that an increase in either quantity results in an increase in the product and the absence of either quantity results in the vanishing of the product. In contrast to intensive measures, product measures clearly depend on the amounts of the constituent quantities.

Consider, for example the following problem:

A cleaning and refurbishing company is confronted with these data:

Name of job	# people available	Nature of job	Estimated time - hours	Other needed information??
Joe's yard	1	Remove debris	30	
Sally's attic	2	Haul out boxes	18	
Sam's basement	3	Remove debris & clean	11	
Rose's storeroom	4	Remove boxes – rearrange contents	8	
School locker rooms	5	Clean out and paint lockers	7	
Town archives	6	Sort/keep/discard old records	6	

The company needs to provide each of their potential clients with a cost estimate. What additional information might they want to have? Can you devise a measure of "size of task" that would allow this company to order these tasks from "smallest" to "largest"? Could your measure be used to determine the "size" of any future task that the company may be asked to do?

Here is a way we might approach the problem using a product measure:

> We might construct a measure of "size of task" by multiplying the number of people available for a task by the number of hours the task will take. The measure of the "size of task" is in person-hours and results in the following

Joe	30 person-hrs
Sally	36 person-hrs
Sam	33 person-hrs
Rose	32 person-hrs
School	35 person-hrs
Town	36 person-hrs

> If the company charges $10./person-hr [a quotient measure] then we can construct yet another product measure of the "size" of the job resulting in

Joe	$ 300
Sally	$ 200.
Sam	$ 330.
Rose	$ 320.
School	$ 350.
Town	$ 360.

We may find this sort of measure a little simplistic because it does not take into account some of the fixed costs attached to doing business. This leads to the possibility of yet another kind of measure which can additively aggregate quotient and product measures.[15]

> Suppose we charge $10 per person and $ 15 per hour. Then our measure of "size of task" in $ is equal to

> $10/person • {number of people} + $15/hour • {number of hours} resulting in the following:

Joe	$ 460.
Sally	$ 290.
Sam	$ 195.
Rose	$ 160.

[15] It should be pointed out that this measure of "size of task" discussed above has a structure that is often used in the social sciences – i.e. $\alpha_1 A_1 + \alpha_2 A_2 + ... + \alpha_n A_n$ where the set {A_i} denotes the set of attributes that are deemed to be important for the composition of the measure [{number of people} and {number of hours} in the case written about above] and the set of coefficients {α_i} are used to make the measure dimensionally coherent [{$/person} and {$/hour} in the case written about above] and also to express the relative importance of the different attributes in composing the measure.

School	$ 155.
Town	$ 150.

The first thing to note is that the two measures do not produce the same ordering of "size of task". Moreover, it is clear that many other possible measures are possible. For example, we might choose to take into account the per hour cost of different employees, the degree of hazard attached to the nature of the job, the time of day during which the work might be done, etc.

Nonetheless, these two measures are reasonably simple. Are they useful? Is there a context in which we might be able to say that one measure is preferable to the other? The reader is invited to formulate other measures of "size-of-task".

Let us examine another example of a product measure. Suppose we are interested in devising a measure of how difficult it might be to stop a moving object[16]. Let us reason in a way similar to the previous case in which we devised a measure of density.

> *Assume we have a collection of objects (say, pickup truck, fast pitched baseball, car, motorcycle, bicycle, etc.) that are moving. Most people would be willing to agree that there is some property of a moving pickup truck that is usually "larger" than that of a moving bicycle. Suppose, for the moment, we call this property "oomph"*
>
> *Is there some way we can pin down what such a property called "oomph" might be? More to the point, is there some way we can say whether the "oomph" of some other moving object is closer to the "oomph" of the pickup truck than it is to the "oomph" of the bicycle? And if so, how much closer?*
>
> *More generally, if we have a bunch of different moving objects, can we find a way to order them according to the "oomph" property – from the one with the "largest" to the one with the "smallest"?*
>
> *First thoughts would suggest that the relevant properties of the moving objects that we want to pay attention to are how heavy they are and how fast they are moving. Each of the moving objects has both a mass and a speed.*
>
> *Assuming we have the proper instruments for measuring speed and mass, we could measure both the speed and the mass for each of the moving objects. If we do so, we will quickly discover that our mystery property, "oomph", cannot be mass and it cannot be speed. A baseball, which is clearly less massive than a car, can be much harder to stop that a slowly rolling car. By the same token, even a very slowly rolling locomotive can be much harder to stop than a fast moving car [a fact that can be attested to by many drivers injured in attempting to beat a train across a grade crossing].*
>
> *Clearly, any measure of this "oomph" property has to take <u>both</u> speed and mass into account.*
>
> *In order to formulate a measure of "oomph" we could try to "add" measures of speed and mass. However, doing so leads to the unfortunate result that a body could have "oomph" even if it were standing still. Also, this kind of measure of "oomph" leads to the conclusion that even a particle with no mass can have "oomph"*[17]

[16] The moving objects in this paper are assumed for the sake of simplicity to all be moving in a straight line.
[17] We clearly are omitting from this discussion any consideration of the momentum of photons.

A more successful measure of "oomph" would be a product of mass and speed. This sort of measure avoids the difficulties that the previous proposed measure had. With this measure a moving object has no "oomph" if it is standing still. Moreover, even at a fixed speed, the smaller the mass of the moving object, the smaller its "oomph".

This measure of how difficult it might be to stop a moving object is normally not called "oomph" but is called <u>momentum.</u>

Such product measures abound in the natural sciences.

<u>How are measures formulated?</u>

Can we use the examples we have looked at to describe some of the component mental activities that were involved in the formulation of a measure?

Here is a list of proposed components of the act of formulating a measure

- *observing*
- *comparing*
- *ordering*
- *making measurements*
- *analyzing data*

Observing

First of all, one must identify the attribute or attributes that are candidates that are deemed to be important for incorporation into the measure one seeks to formulate. In the case of the quotient measure *density* these candidates were mass and volume. In the case of the product measure *momentum* these candidate attributes were mass and velocity.

Comparing

Can the amount or degree of presence of an attribute such as volume or mass in one case be compared to the amount or degree of presence of that same attribute in a second case?

Ordering

Can the amount or degree of presence of a given attribute such as volume or mass be ordered across a series of different cases?

Making measurements

Is there a metric that allows the amount or degree of presence of a given attribute to be measured in a variety of cases and or situations?

Analyzing data

Are there regularities that emerge in the analysis of the data that suggest a potentially useful measure? In the case of density, graphical analysis of the data suggests a quotient measure that is proportional to mass and inversely proportional to volume.

Is The Measure “Good Enough”?

Choosing a measure to characterize the state of a system is an exercise in deciding what features of the system you want to include in your characterization and what features you deem sufficiently unimportant for the purpose at hand that they may be neglected, at least at first.

A measure of density may not, at first, contain any reference to temperature. A measure like momentum may contain at first a reference only to the speed of the center of mass of a body and not refer to the distribution of mass and velocity within the body.

Thus the process of formulating a measure as a first step on the road to modeling is of necessity a tentative one. Ultimately, the question of “is the measure good enough” can only be answered by listening to the fugue of the predictions of the model formulated with this measure and the purpose for which the model has been formulated.

From Measures to Models

Measures that are models & measures that are not models

Are measures models? That depends. In my view an essential property of a model is its ability to predict the value of a measure in an *as yet untried instance*.

In the case of density discussed above, we designed a measure (and called it density) based on data collected from a variety of objects that were fashioned from a variety of materials.

Can we predict the value of the density measure for an object whose mass and volume we have not previously measured? Under limited circumstances the answer is yes. If the material is one with which we have worked before then using our rule for how mass and volume are related for the material in question -

if we measure the mass we can predict the volume
if we measure the volume we can predict the mass
if we measure the mass and the volume we can predict the ratio of mass to volume[18]

[18] It has to be stressed that for a finite number of data points the inferred linear mathematical relationship between mass and volume is *not* unique. The assumption of a linear relationship is part of our assertion about the way nature behaves in the *as yet untried instances*.

On the other hand, if we have not worked with the material before we cannot make such predictions.

In contrast, consider "-ness" measures such as "square-ness". Despite the fact that the task was presented with a finite number of rectangles we did not simply infer a rule for quantifying "square-ness" from these few rectangles. This is in contrast to the density case where we were dealing with real objects made from real materials and we were obliged to devise our rule from limited data. In the case of rectangles, we were able to take ***all*** rectangles into account in formulating our measure. As a result there are no *as yet untried instances*. Thus from my perspective, "square-ness" is not a model – it is simply a measure.

In the case of our measure of difficulty of stopping a moving object we have a related situation. We have asserted that the value of the momentum of *any* moving object can be calculated from knowing the mass and the speed of that object. There are no *as yet untried instances*. Momentum, by itself, is not a model – it is a measure.

Getting from Measures to Models

The centrality of "as yet untried instances" to this discussion is by now evident. It is a manifestation of the need for models to be falsifiable[19]. A measure whose value may always be calculated from its definition is thus clearly not a model.

Does this mean that measures are not germane to a discussion of models? Not at all. Measures are the constructs we need to formulate in order to build models. A model in physics, for example, may predict how a measure like momentum will change with time. A model in chemistry may predict how a measure like concentration of a species might change with time. A model in economics might predict how a measure like gross national product might change with time. A model in environmental science might predict how a measure like average global temperature might change with time.

Measures can vary with parameters other than time – e.g., measures such as density and pressure in gases can vary with temperature. The essential point is that in order to have a model we must be able to predict the values of a measure that we are interested in instances that did not enter into the formulation of that measure.

Ideally, one would like to predict how the values of the measure one formulates might vary with time or temperature or some other parameter(s) of interest. However, we may not understand the underlying science well enough to be able to make such predictions. We may, however, have a qualitative understanding of underlying mechanisms that is good enough for us to say the value of our measure will increase or decrease with time or

---

[19] See for example Popper, Karl, *The Logic of Scientific Discovery*, Basic Books, New York, NY, 1959.

temperature. Such semi-quantitative models have been the subject of some considerable investigation[20],[21]

<u>Agent-based models – from local interactions to global measures of complex behavior</u>

The explosive growth of computational power has enabled us to approach the problem of modeling phenomena of all sorts in a fashion that follows precisely the opposite sequence to that described up until now in this paper. These models are called "agent-based" models and are built on the assumption that phenomena can be simulated by considering a large number of agents that interact with one another in space and time via interactions that can be stated and computed simply. Although the possibility of such models was first described in the 1940s it was not until the widespread availability of computers that it became realistic to model phenomena in this fashion.

Here is a quote from a description of agent-based models[22]:

The situatedness of the agents and their responsive, purposeful behavior are encoded in algorithmic form in computer programs. The modeling process is best described as inductive. The modeler makes those assumptions thought most relevant to the situation at hand and then watches phenomena emerge from the agents' interactions. Sometimes that result is an equilibrium. Sometimes it is an emergent pattern. Sometimes, alas, it is an unintelligible mangle.

Here is an example drawn from an agent-based model of fish and shark populations[23] in a rectangular grid:

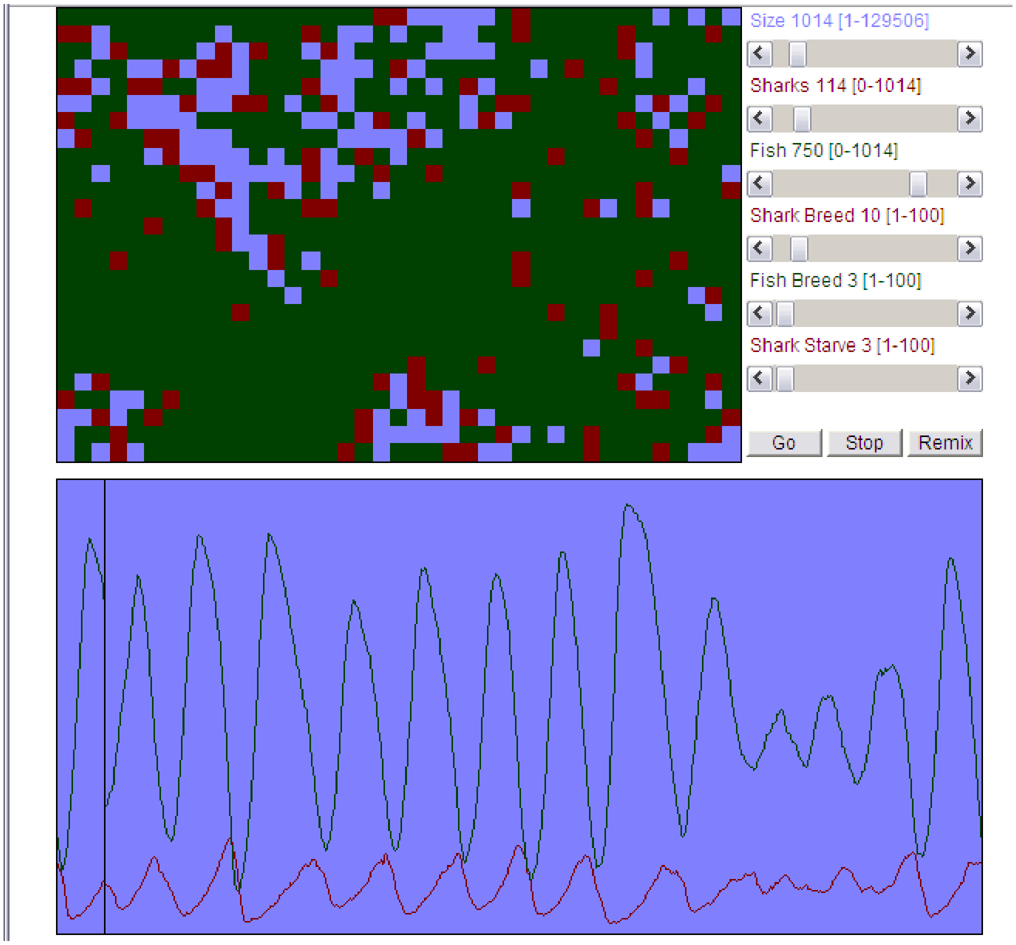
---

[20] See for example - Miller, R., Ogborn, J., Briggs, J., Brough, D., Bliss, J., Boohan, R., Brosnan, T., Mellar, H., & Sakonidis, B. (1993). Educational tools for computational modelling. Computers in Education, 21(3), 205-261.

[21] See also Jackson, S., Stratford, S. J., Krajcik, J., & Soloway, E. (1995a). Making system dynamics modeling accessible to pre-college science students. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

[22] http://en.wikipedia.org/wiki/Agent_based_model

[23] http://www.leinweb.com/snackbar/wator/



The upper part of the illustration shows the location of fish and sharks at a particular instant in the evolution of the system. The lower part of the illustration shows two graphs, one of the fish population as a function of time and the other the shark population as a function of time.

Despite the fact that the rules governing the behavior of individual fish and individual sharks are quite simple we see that quite complex oscillatory behavior can emerge when the model is run. Here is a short summary of the rules[24].

> *The program is dependent upon five parameters, plus the size of the rectangular grid. The first parameter is the number of fish, the prey. A fish swims at random to one of the four horizontally or vertically adjacent spaces on the grid, if it is unoccupied. … The second parameter is the fish breed time. If a fish survives this number of cycles and an open space is available, a new fish is bred. The third parameter is the number of sharks, the predators. A shark eats a fish in an adjacent space on the grid. A shark swims to an open space if there is no adjacent fish. The fourth parameter is the shark starve time. If a shark finds no fish for this number of cycles, it*

[24] *ibid*

> *dies. The final parameter is the shark breed time. A shark breeds in the same way as a fish.*
>
> *If the number of sharks is very small, the fish population will increase quickly and the shark population will soon follow because they find plenty of food. Eventually the fish will become overcrowded and overwhelmed by the large shark population and the fish population will crash. Depending upon the quantity and distribution of sharks at the beginning of the crash, the fish may disappear entirely, the sharks may become isolated and disappear or catastrophe may be narrowly averted.*

I cite this example in some detail to illustrate the fact that the statement of the individual fish and shark behaviors is the starting point of this model. The - as yet unexamined – entities in this model are the global measures[25] that emerge only after the individual fish and shark behaviors are allowed to develop. This is in contrast to the kind of model discussed earlier in which measures are formulated and then their time or other parametric behaviors are predicted.

Some implications for K-12 science and mathematics

The word “model” is widely used and, in my opinion, widely misused. By introducing this distinction between measures and models, it is my hope that explicit attention can be paid in K-12 science and mathematics classes to the formulation of measures. This in itself is a worthy effort that deserves attention as is apparent from even a brief consideration of some of the Balanced Assessment “–ness” tasks described above.

Moreover, separating the task of formulating a measure from that of formulating a model that uses that measure focuses attention on the distinction between the measure construct and the model construct. This is likely to result in a better understanding on the part of teachers and students of the differences and similarities between mathematics and science. But that is the subject of another discussion!

---

[25] Some of these measures include equilibrium states, frequency, amplitude and relative phase of the oscillatory behavior of populations, conditions for collapse of one or both species, etc.